Black Holes to Algebraic Curves: Consequences of the Weak Gravity Conjecture

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Based on 1509.06374/hep-th, 1606.08437/hep-th, 1906.02206/hep-th with Ben Heidenreich and Matt Reece, work to appear with Murad Alim and Ben Heidenreich

Outline

- The Landscape and the Swampland
- The Weak Gravity Conjecture
- A WGC Counterexample?
- Black Holes and BPS Bounds
- Mathematical Implications

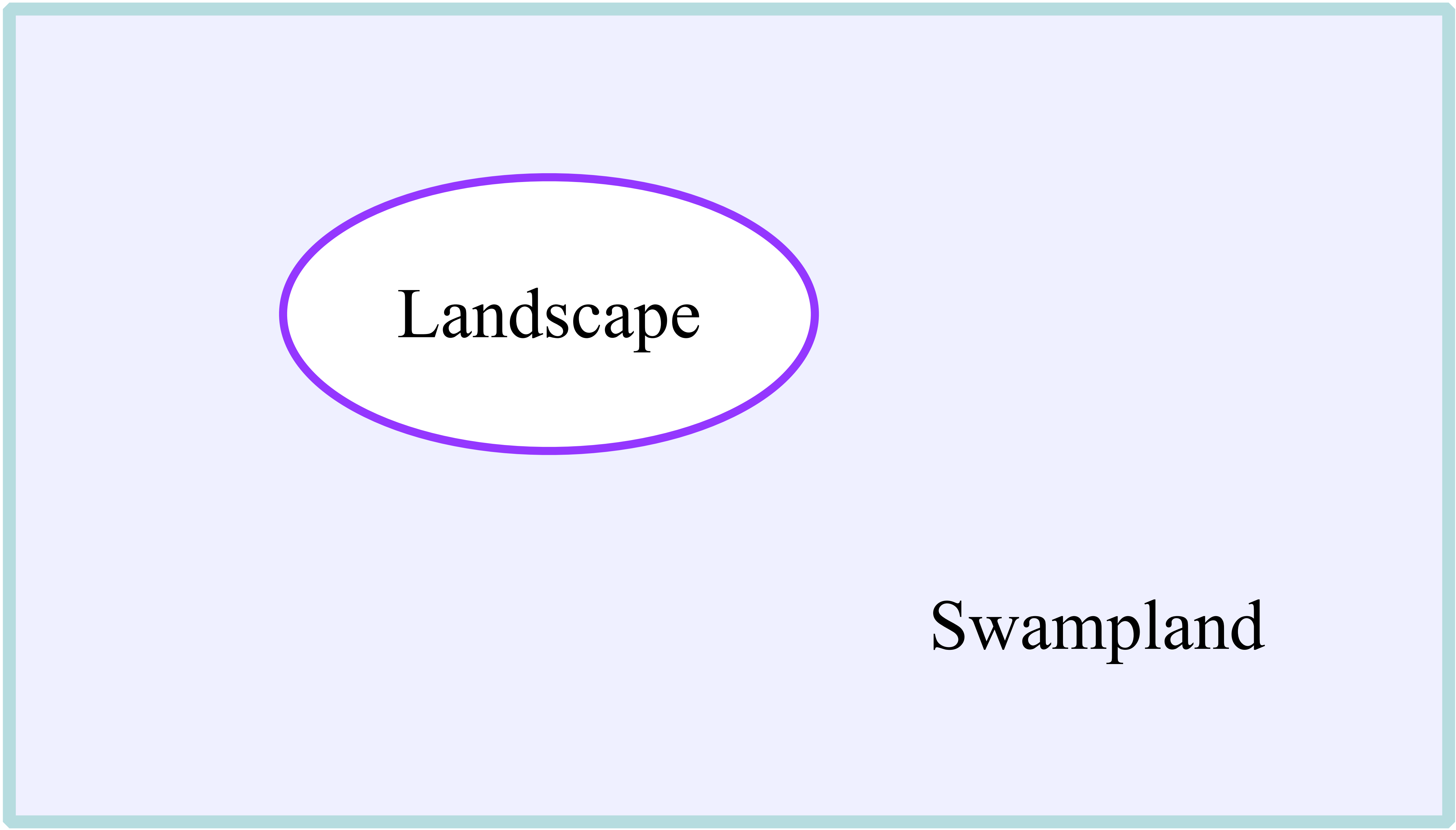
The Landscape and the Swampland

The Landscape

The Swampland

Vafa ’05, Ooguri, Vafa ‘06

Goal: Delineate Boundary



The Weak Gravity Conjecture

Weak Gravity Conjecture (WGC)

In any d-dimensional U(1) gauge theory coupled to quantum gravity, there must exist a "superextremal" particle of charge q, mass m, with

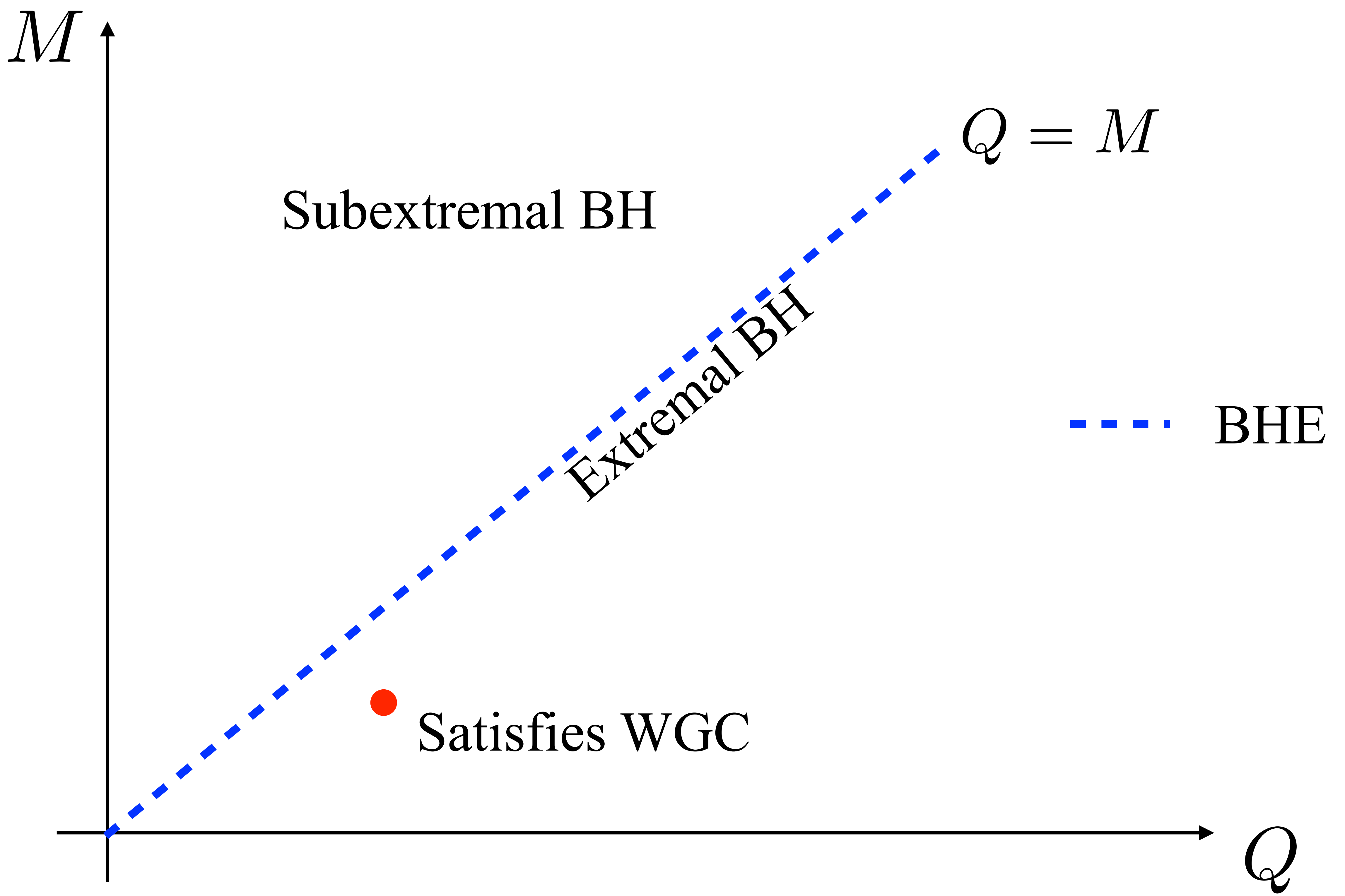
$$\frac{q}{m} \geq \frac{Q}{M}\Big|_{\rm ext} = \frac{\gamma_d}{M_{\mathrm{Pl};d}^{(d-2)/2}} \qquad (q = en)$$

charge quantum (n)

coupling constant (e)

Arkani-Hamed, Motl, Nicolis, Vafa, '06

WGC vs. Black Hole Extremality



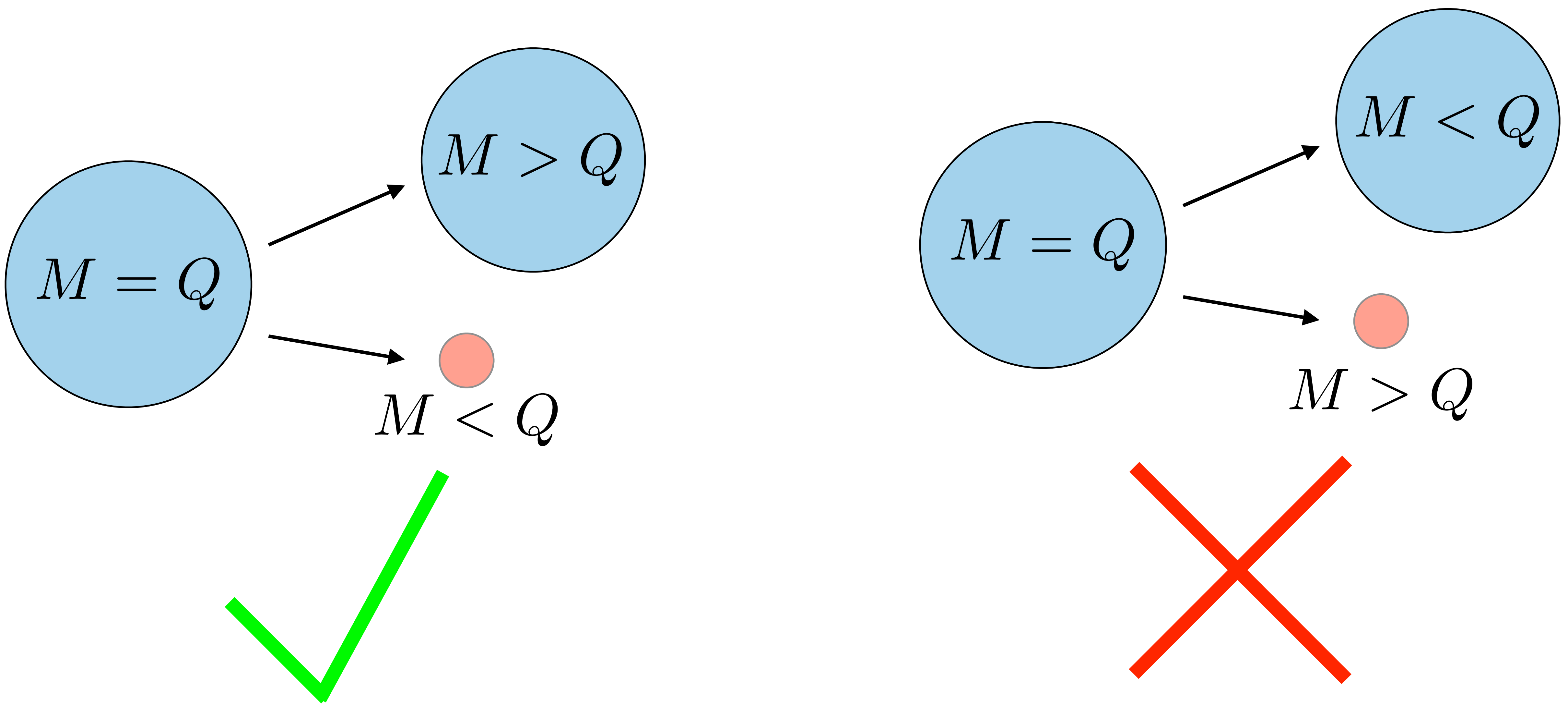
The WGC and Electromagnetism

$$F_{\mathrm{EM}} \sim \frac{q_e^2}{r^2} \qquad F_{\mathrm{grav}} \sim \frac{m_e^2}{r^2}$$

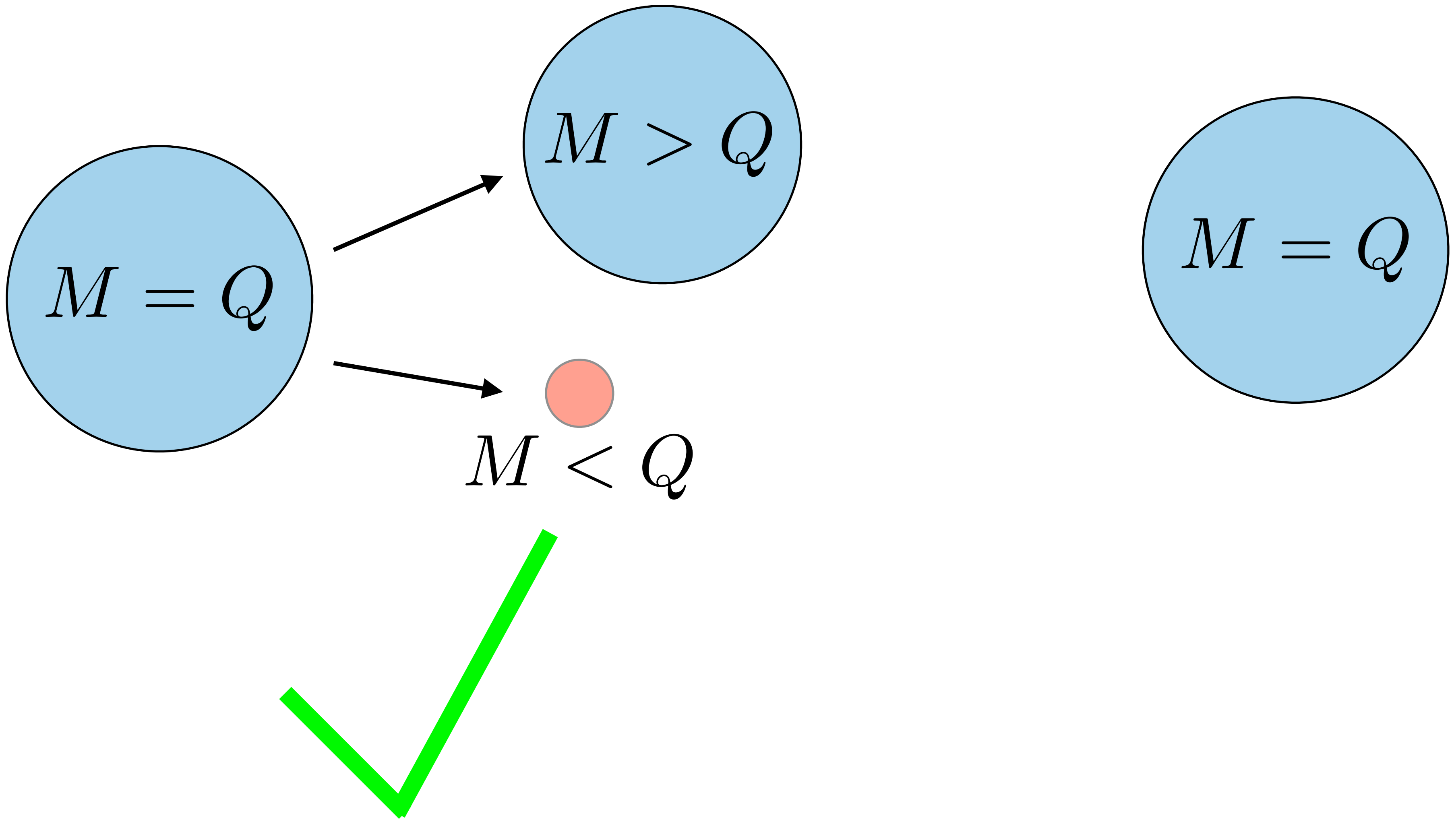
$$m_e \sim 10^{-21} M_p \qquad q_e \sim 0.1$$

$$\Rightarrow \quad \frac{q_e}{m_e} \gg \frac{1}{M_p} \quad , \quad F_{EM} \gg F_{\mathrm{grav}}$$

Motivation for the WGC



Motivation for the WGC



Evidence for the WGC

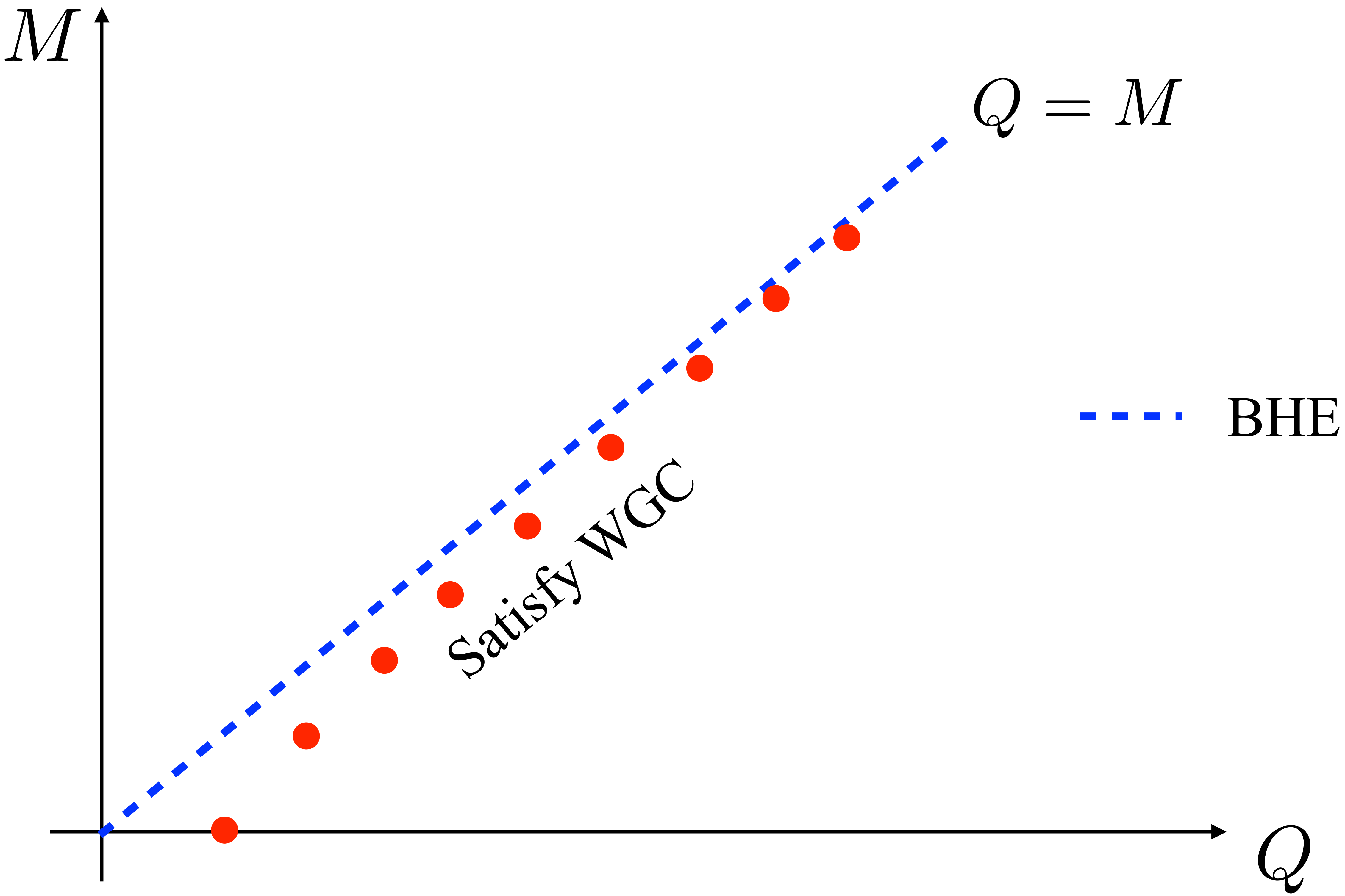
- *Many* string theory examples Arkani-Hamed et al. ’06; Heidenreich, Reece, T.R. ‘16
- Black hole entropy arguments Fisher, Mogni ’16; Cheng, Liu, Remmen ‘18
- Cosmic censorship argument Crisford, Horowitz, Santos ’17; Horowitz, Santos ’19
- Holographic arguments Harlow ’15; Urbano ’18; Montero ‘18
- Scattering arguments Cheung, Remmen ’14; Hamada, Noumi, Shiu ’18; Arkani-Hamed, Huang ‘19
- Relaxation argument Hod ‘17

A WGC Inconsistency

- Consider *(d+1)*-dimensional U(1) theory satisfying WGC
- KK reduce on small circle
- Find violation of WGC in *d* dimensions!

Heidenreich, Reece, T.R. '15

Towers of Superextremal Particles



Variants

- Superextremal particles at:
  - Infinitely many sites in the charge lattice (Tower WGC)   Andriolo, Junghans, Noemi, Shiu '18
  - Every charge site in a sublattice (sLWGC)
  - ~~Every site in the charge lattice (LWGC)~~

Heidenreich, Reece, T.R. '16

$$\mathrm{WGC}_d \overset{\sim}{\Rightarrow} \mathrm{T/sLWGC}_{d+1}$$

Phenomenological Implications of a Strong WGC

- Axion decay constants sub-Planckian AMNV ’06, T.R. ’14, ’15, Brown, Cottrell, Shiu, Soler ‘15
- Photon exactly massless Reece ‘18
- QCD axion decay constant below GUT scale Heidenreich, Reece, T.R. '16
- Chromonatural inflation incompatible with GUTs Heidenreich, Reece, T.R. '17
- …

Evidence for the T/sLWGC

- String orbifold compactifications Heidenreich, Reece, T.R. ’16
- Perturbative string theory—follows from modular invariance Heidenreich, Reece, T.R. ’16; Montero, Shiu, Soler ’16
- Certain classes of BPS states in IIA compactifications Grimm, Valenzuela, Palti ’18
- F-theory compactifications to 4d/6d Lee, Lerche, Weigand ’18, ’18, ‘19
- Scalar field theories, by infrared consistency Andriolo, Junghans, Noemi, Shiu ‘18

A T/sLWGC Counterexample?

BPS States

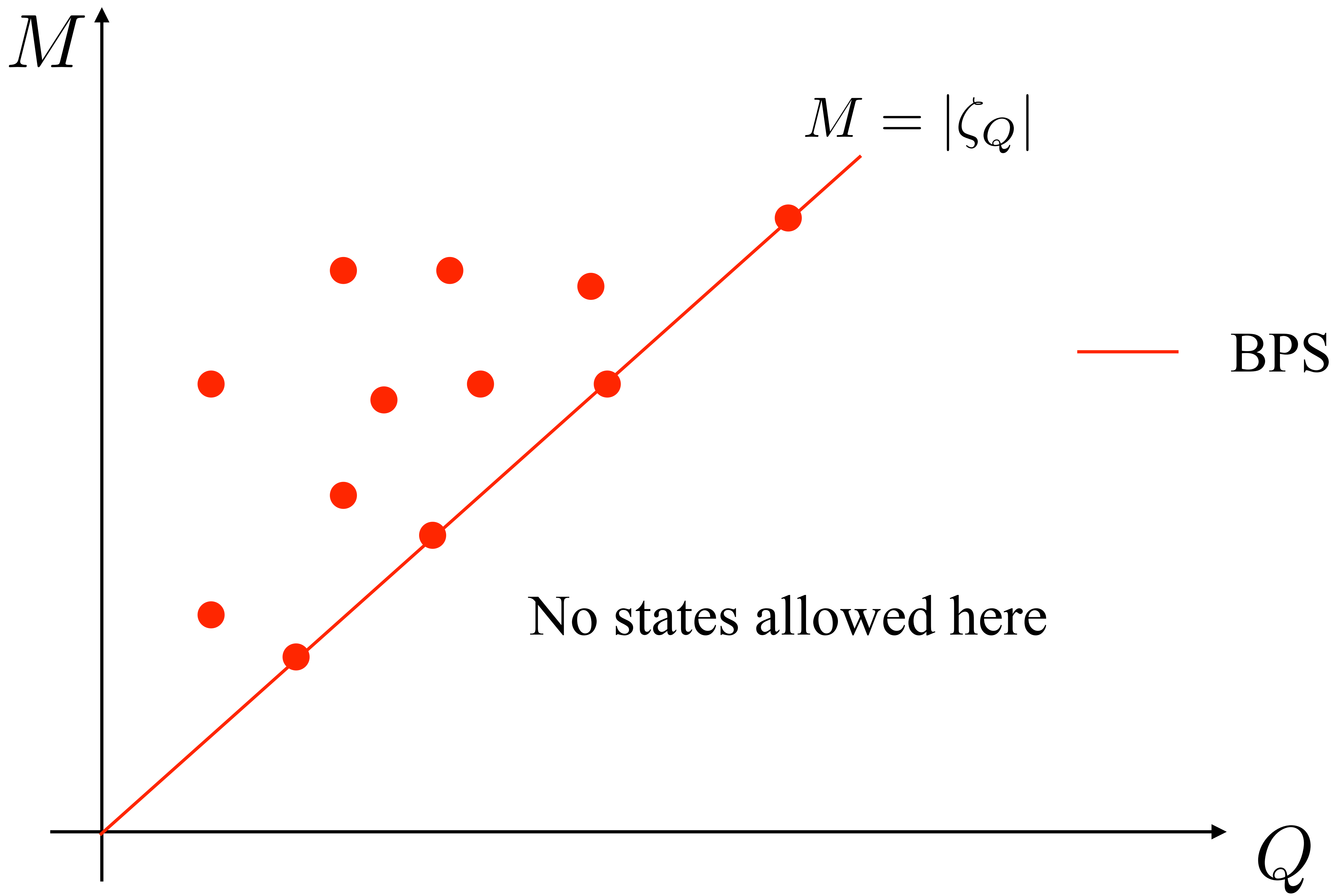
- In theories with 8+ supercharges, charged states must satisfy BPS bound:

$$m \geq |\zeta_{q_i}(a_i)|$$

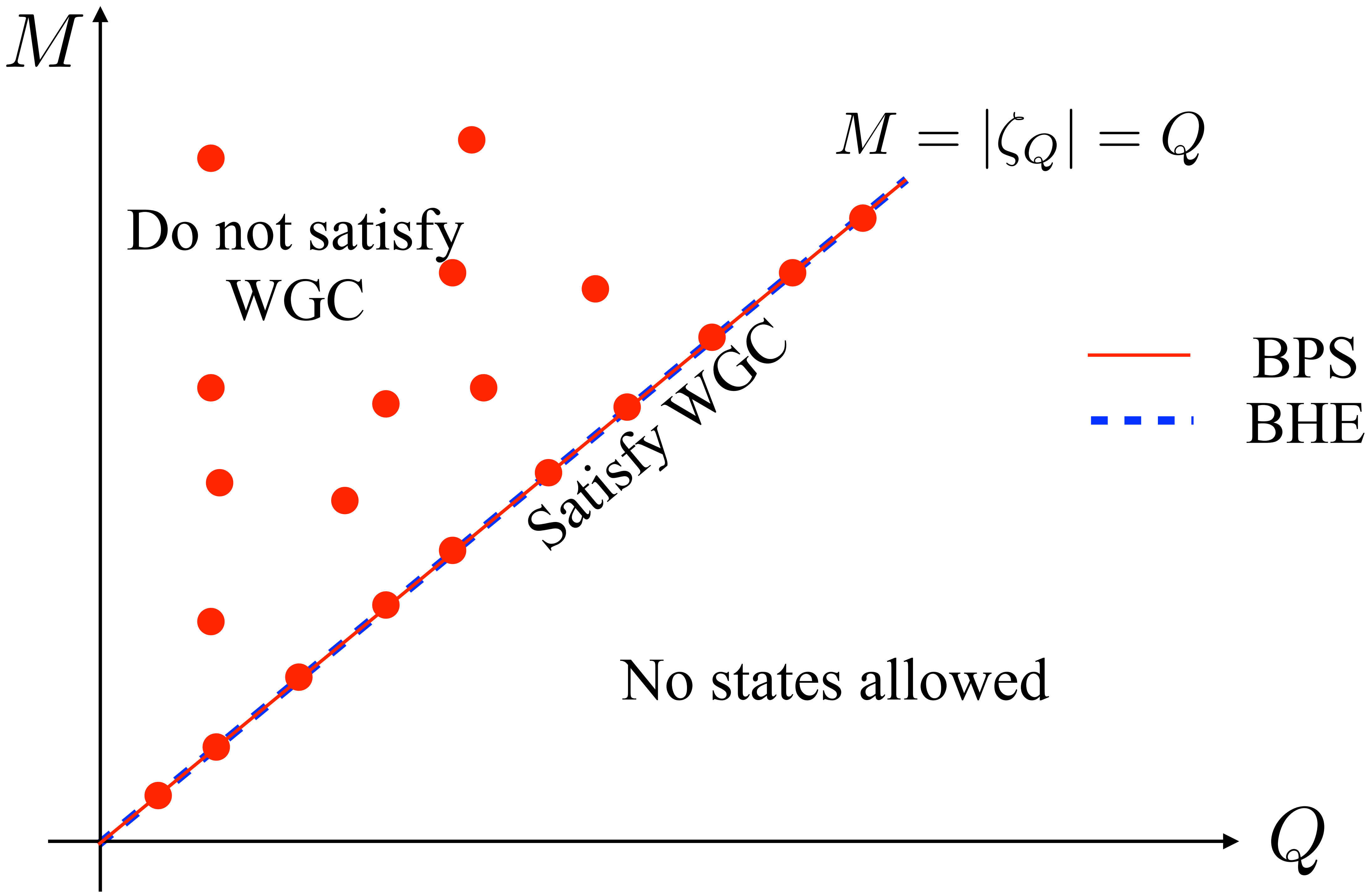
- Central charge ζ depends on moduli a_i, charge q_i of state
- 4d: ζ complex. 5d: ζ real
- States that saturate the BPS bound are called BPS states
- In 5d theories from M-theory on CY_3, M2-brane BPS states counted by Gopakumar-Vafa invariants

Gopakumar, Vafa, '98

BPS Bound



BPS vs. WGC



BPS = BHE ⇒ Only BPS states satisfy WGC!

A T/sLWGC Counterexample?

- At conifold singularity in IIA/M-theory, only states of charge q = 1 become massless:

$$m_{q=1} \to 0, \qquad m_{q\neq 1} \not\to 0.$$

Strominger '95

- No tower of BPS states!
- GMSV conifold:

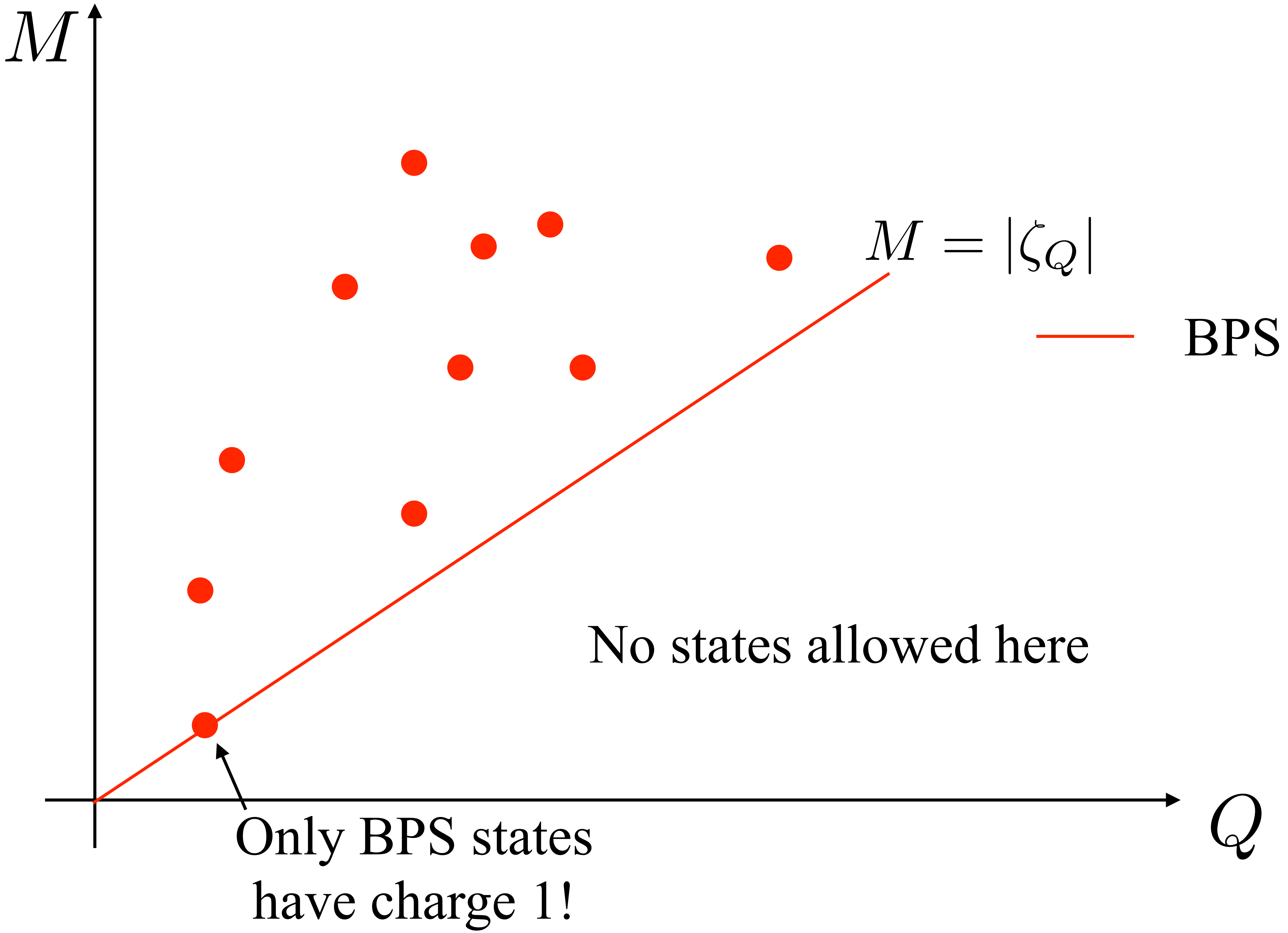
Greene, Morrison, Strominger '95

Greene, Morrison, Vafa '96

q_2 \ q_1	0	1	2	3	4
0	–	640	10032	288384	10979984
1	16	2144	231888	23953120	2388434784
2	0	120	356368	144785584	36512550816
3	0	−32	14608	144051072	115675981232
4	0	3	−4920	5273880	85456640608
5	0	0	1680	−1505472	3018009984
6	0	0	−480	512136	−748922304
7	0	0	80	−209856	218062416
8	0	0	−6	75300	−90910176
9	0	0	0	−21600	37721680
10	0	0	0	4312	−15086208
11	0	0	0	−512	5300736

Empty "wedge" without BPS states

BPS States in the Conifold



Black Holes and BPS Bounds

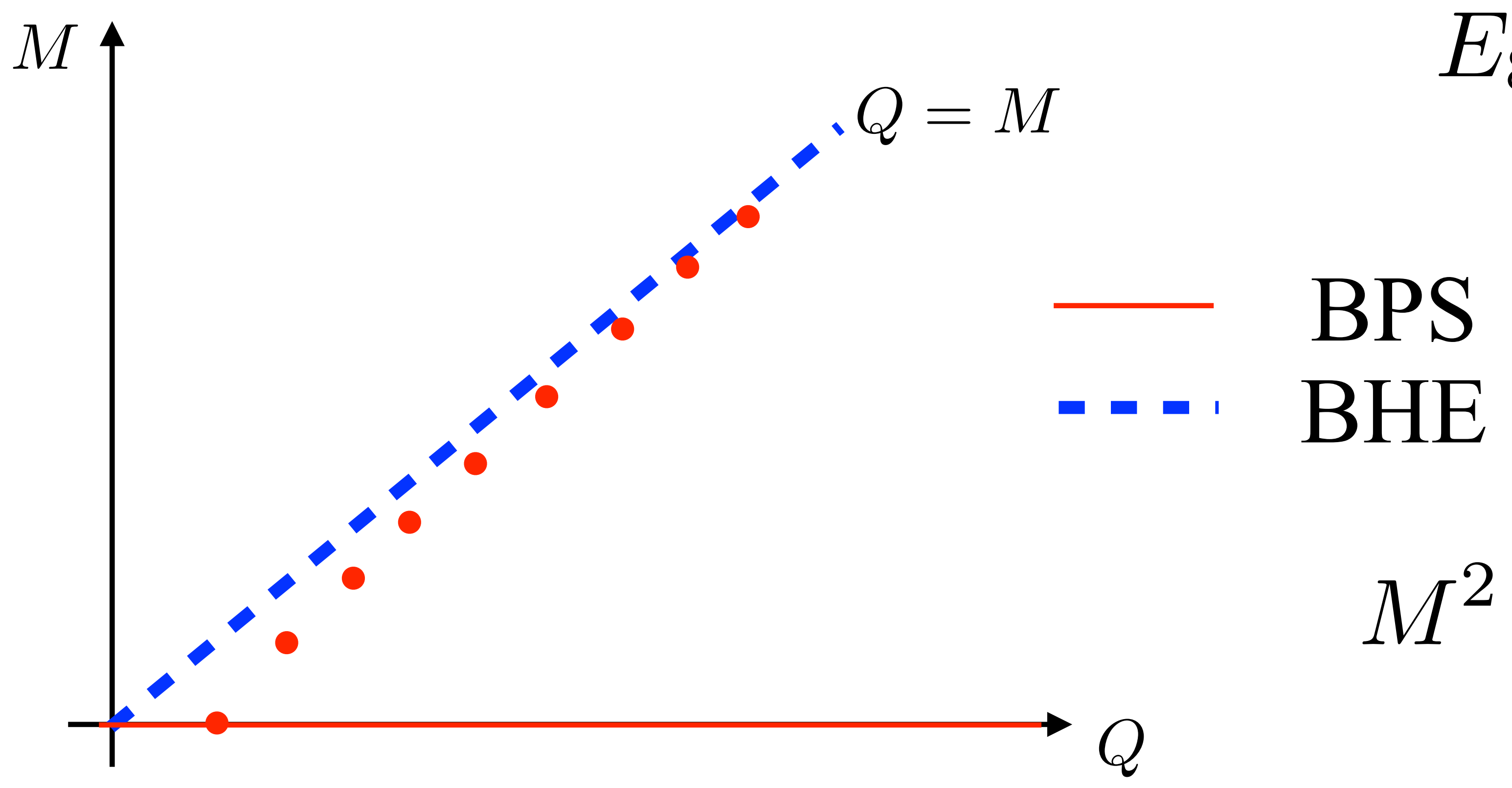
An Important Reminder

- In general,

$$\text{BPS} \neq \text{BHE}$$

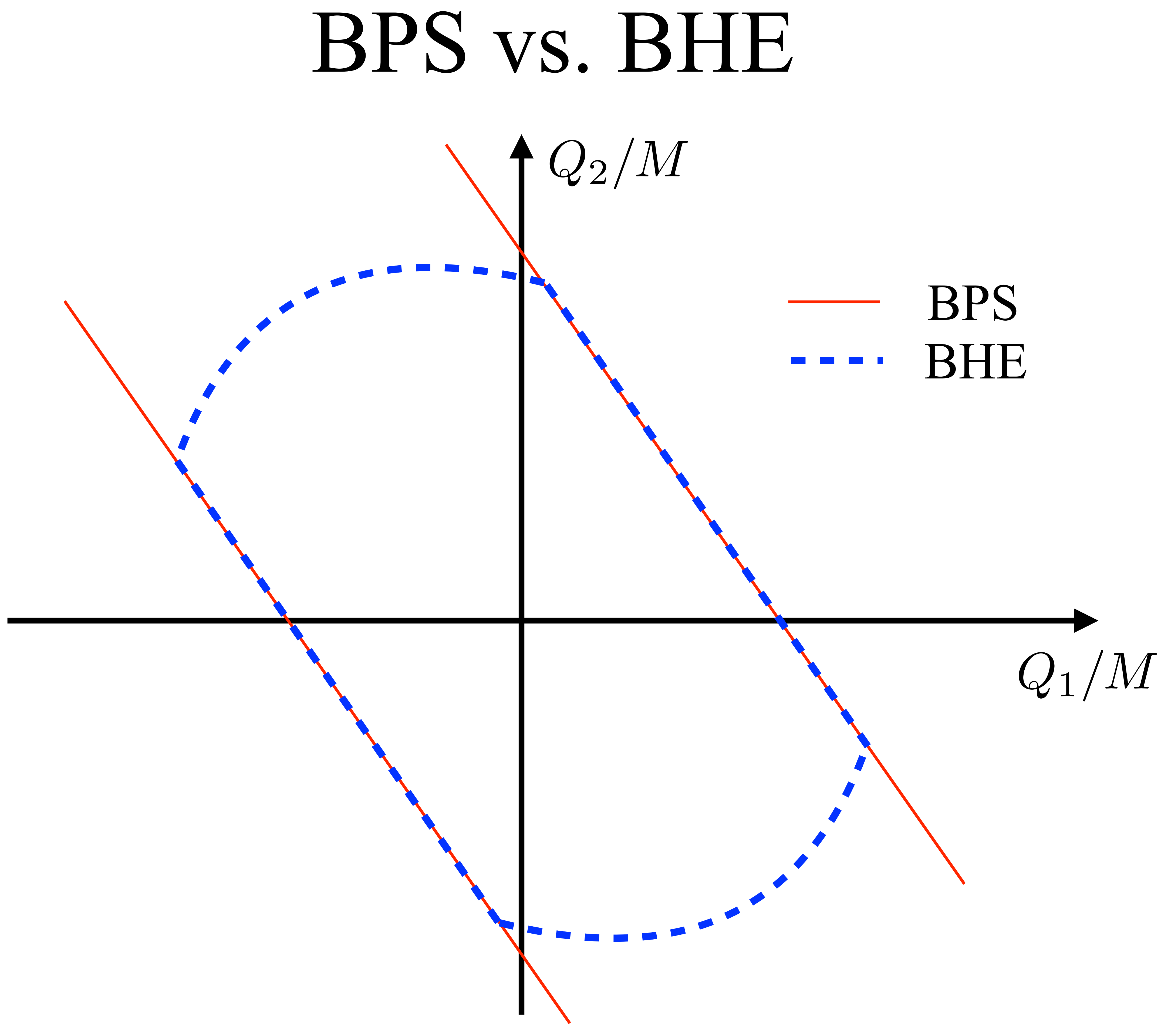
(even in theories with BPS states)

- Ex: heterotic string on torus



$$E_8 \times E_8 \to U(1)^{16}$$

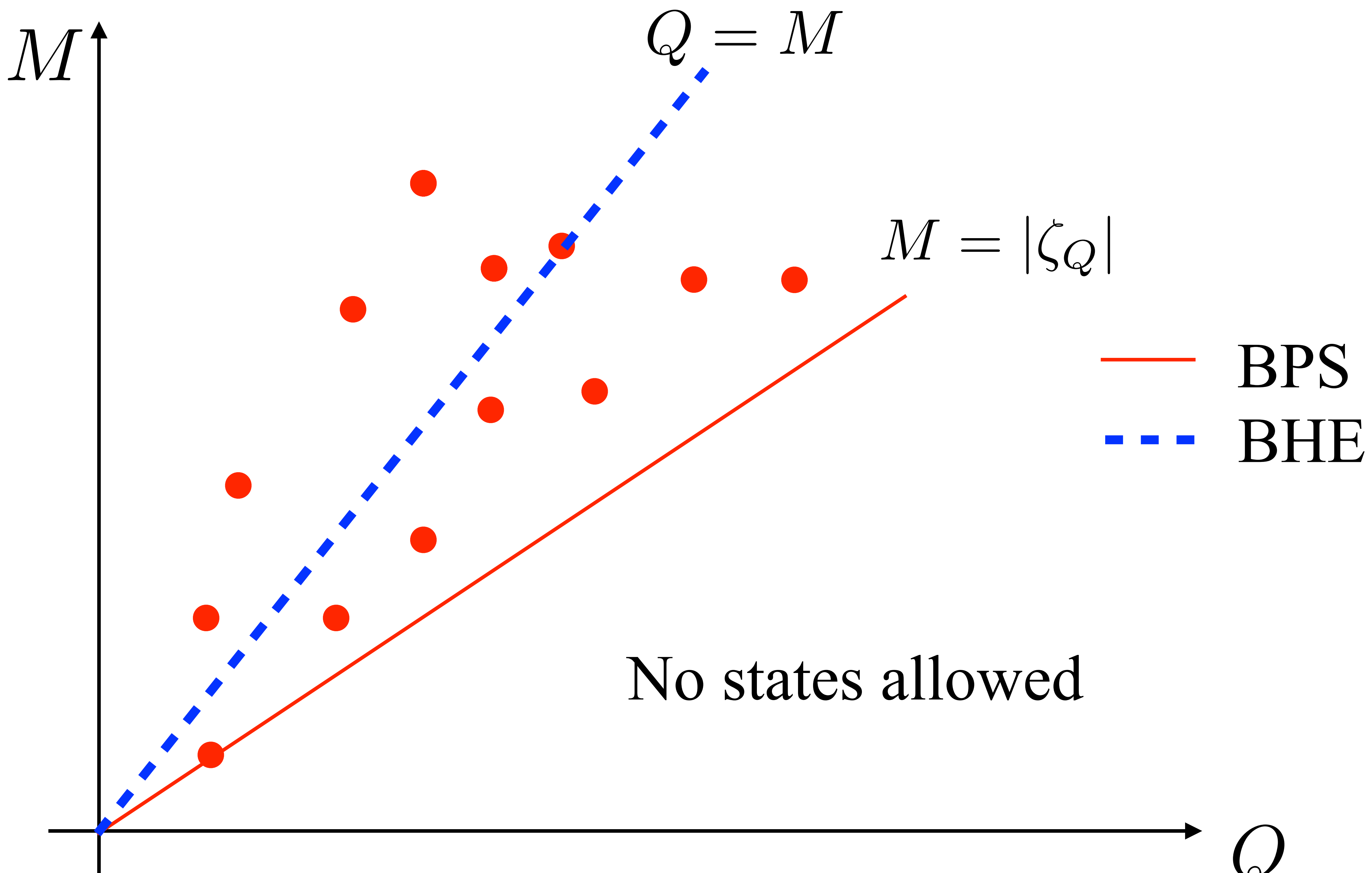
$$M^2 = Q^2 - 1$$



BPS vs. BHE
Q_2/M
BPS
BHE
Q_1/M

BPS vs. BHE

Since $g_5 \not\to 0, \zeta \to 0$, must have BPS $\neq$ BHE for conifold:



BPS vs. BHE

- Q: When does BPS = BHE?

BPS vs. BHE

- Q: When does BPS = BHE?
- A: BPS = BHE $\Rightarrow \zeta_{q_i}(a_i) > 0 \, \forall a_i$

  $\Leftarrow \zeta_{q_i}(a_i)$ has a critical point somewhere in moduli space

BPS vs. BHE

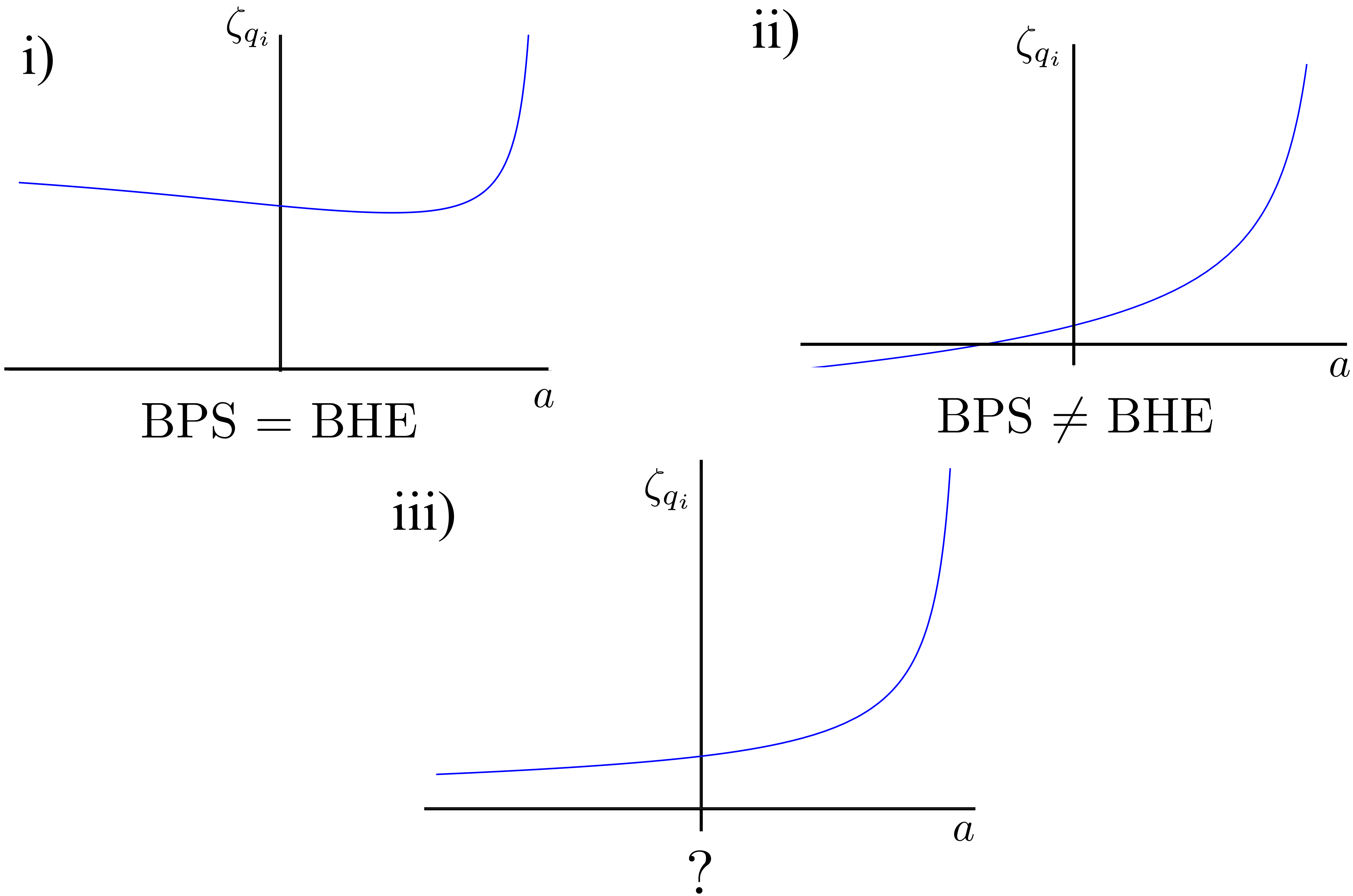
- Proof (sketch):
  - Critical points of ζ_{q_i} are all local minima (maxima) with $\zeta_{q_i} > 0\,(\zeta_{q_i} < 0)$.
  - By Morse theory (+ additional constraints from 5d SUGRA), such local minima are global minima
  - So, either:

    i) ζ_{q_i} has a critical point $\Rightarrow \mathrm{BPS} = \mathrm{BHE}$

    ii) $\zeta_{q_i} = 0$ somewhere $\Rightarrow$ no critical points $\Rightarrow \mathrm{BPS} \neq \mathrm{BHE}$

    iii) OR $\zeta_{q_i} > 0\,(\zeta_{q_i} < 0)$ everywhere, but no critical points

BPS vs. BHE



BPS vs. BHE

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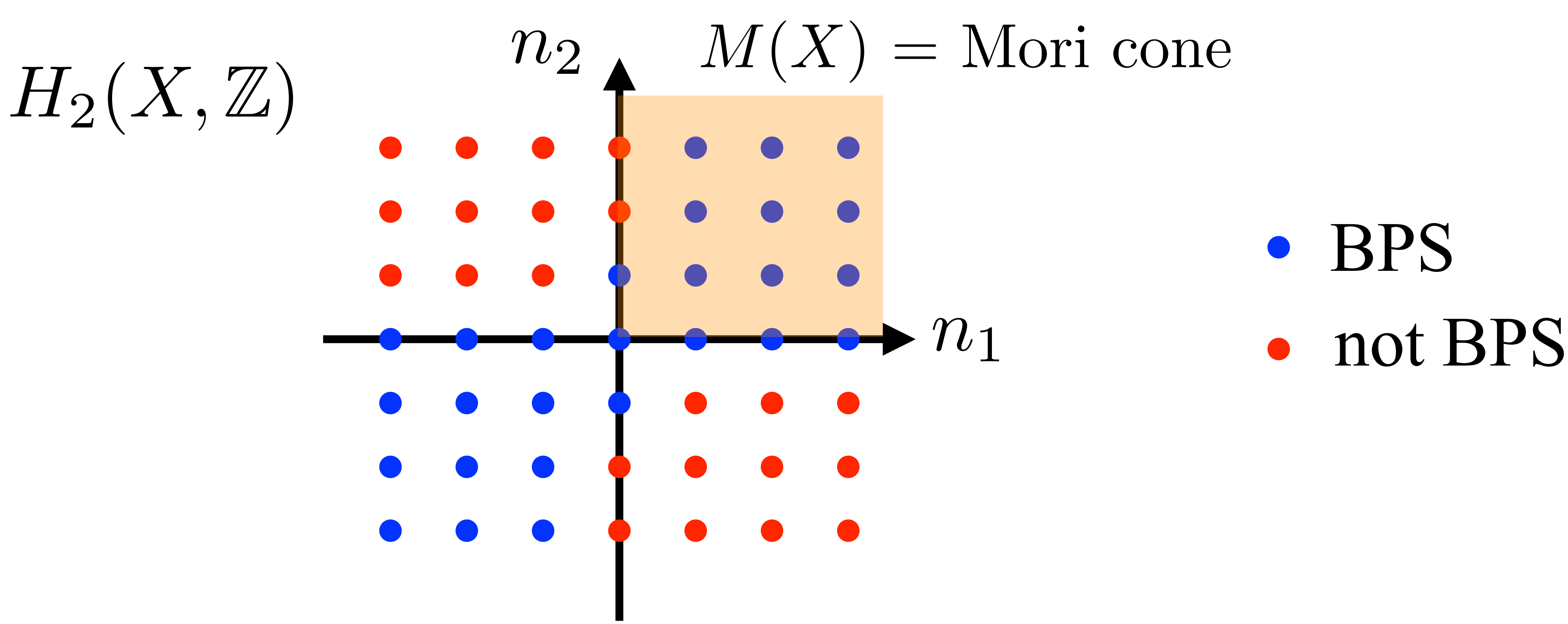
Mathematical Implications

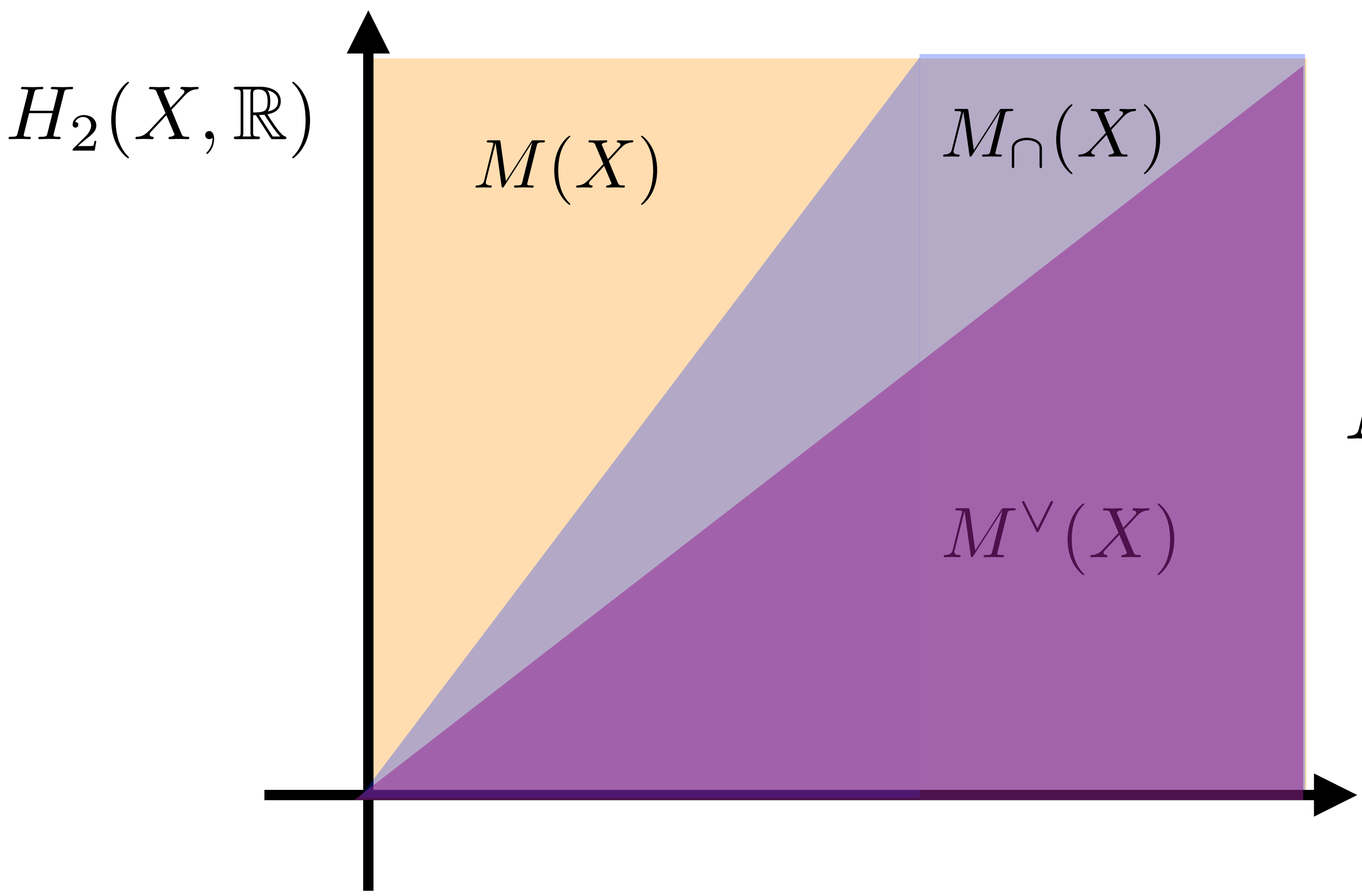
From Algebraic Geometry to Physics

- M2-brane wrapped on holomorphic curve of class $\sum n_i[C_i], n_i \geq 0$ gives BPS state of charge n_i
$\Rightarrow H_2(X,\mathbb{Z})$ identified with electric charge lattice



Three Cones of Interest

- $M(X)$ = Mori cone
- $M_{\cap}(X)$ = dual to extended Kähler cone
- $M^{\vee}(X)$ = dual to effective cone



$$M^{\vee}(X) \subset M_{\cap}(X) \subset M(X) \subset H_2(X,\mathbb{R})$$

SUGRA and Geometry

- $\sum n_i[C_i] \in$ dual to effective cone, $M^\vee(X)$

  $\Leftrightarrow \zeta_{n_i}(a_i)$ has a minimum

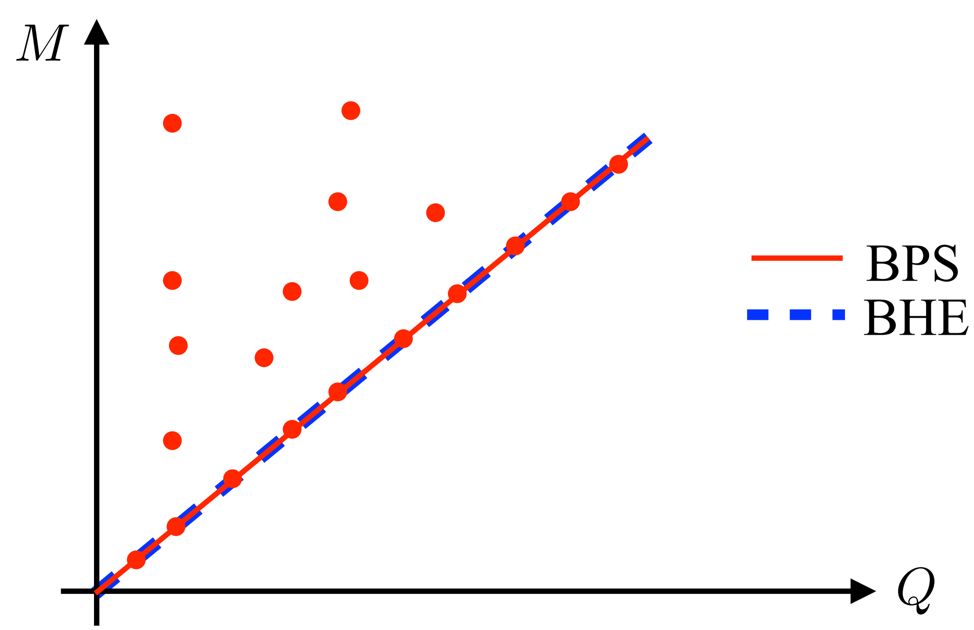
  $\Rightarrow$ BPS = BHE

- BPS = BHE $\Rightarrow \zeta_{n_i}(a_i) > 0$ for all a_i

  $\Leftrightarrow \sum n_i[C_i] \in$ dual to extended Kähler cone, $M_\cap(X)$

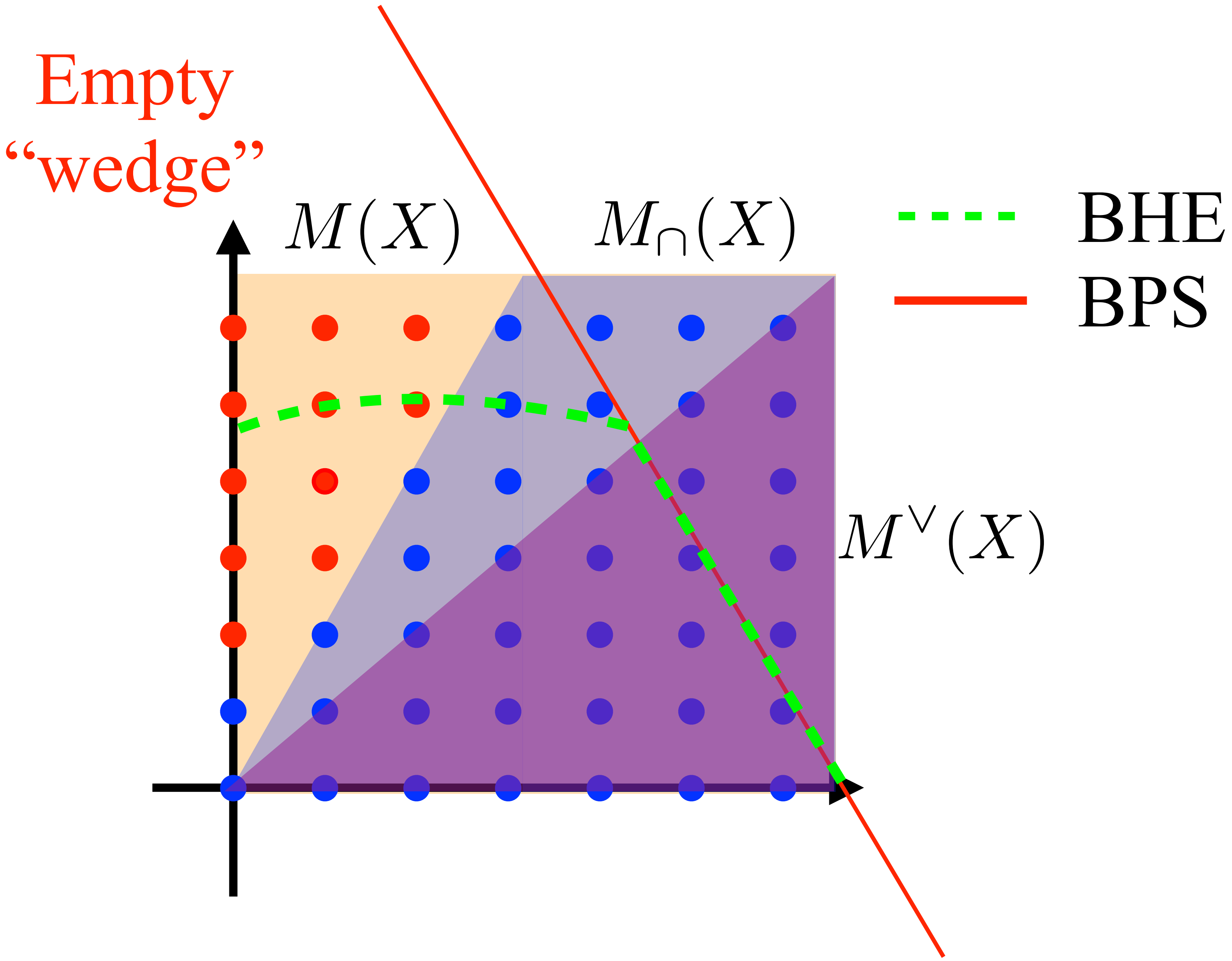
A Geometric Conjecture

- Tower WGC: For all $[C] \in H_2(X, \mathbb{Z})$ in the dual to the effective cone $M^\vee(X)$, there exists a holomorphic curve in the class $n[C]$ for infinitely many $n \in \mathbb{Z}$.



- Conversely, there are only finitely-many holomorphic curves outside the dual to the extended Kähler cone, $M_\cap(X)$.

GMSV Revisited



Summary

- Beautiful interplay between
  - BPS states and GV invariants
  - SUGRA central charges
  - Calabi-Yau geometry/intersection theory
  - Black hole extremality bounds

  leads to consistency with T/sLWGC
- TWGC implies new geometric conjecture, verified in examples
- Other connections between WGC and axion inflation, cosmic censorship, emergence, AdS/CFT, black hole entropy, scattering amplitudes, F-theory, …
- Exciting time to be hiking through the Swampland!