1) Liesuperalgebras + ~~Shortening~~, 2) 4d $N=2$ SCA 3) Short multiplets $\mathcal{E}, \hat{\mathcal{B}}, \hat{\mathcal{C}}$.

hep-th/0209056. Dolan + Osborn.

<u>Def</u>: A Lie superalgebra is a graded vectorspace $g = g_0 + g_1$ equip w/ a bilinear product $\{\cdot,\cdot]: g \times g \to g$ called super Lie bracket.

- g is graded w.r.t. $\{\cdot,\cdot]$

$$\{g_i, g_j] \subseteq \begin{cases} g_0 & i=j \\ g & i \neq j \end{cases} \qquad i,j=0,1$$

$$x_i \in g_i \quad \deg(x_i) = i \in \mathbb{Z}_2$$

g_0 even / bosonic subalgebra $\quad g_1$ odd / fermionic subalgebra

- Bracket Antisymmetry: $x, y \in g$ of definite grading

$$\{x, y] = (-1)^{1+\deg x \deg y}\{y, x]$$

- Super Jacobi $(-1)^{\deg x \deg y}\{x, \{y, z]] + \text{perms} = 0$.

- Even subspace $g_0 \subset g$ is a Lie algebra.
- g_1 is a g_0-representation $\Leftarrow$ ✗

$\mathcal{N}=2$ SCA

$\mathfrak{g} = SU(2,2|2)$ " = Hermitian $(2,2|2)\times(2,2|2)$ Str $=0$

$$\mathfrak{g}_0 = \underbrace{so(4,2)}_{\substack{4d\\ \text{conformal}}} \oplus \underbrace{su(2)_R \oplus u(1)_r}_{R\text{-Sym of } \mathcal{N}=2}$$

generated by $\{M_{\mu\nu}, K_\mu, P_\mu, D\}, \{R_3, R_\pm\}, \{r\}$

$\mu,\nu = 0,1,2,3$

ρ: $\mathfrak{g}_0$-repn labelled by $(j, \bar{j}, \Delta), (R), (r)$

$j, \bar{j}, R = 0, 1/2, 1, 3/2, \ldots, \quad \Delta, r \in \mathbb{R}$

$\mathfrak{g}_1$ generated by

$$\begin{array}{lccccccc} & & j & \bar{j} & \Delta & & R & & r \\ Q^A_\alpha & \in & (1/2, & 0, & 1/2) & \oplus & 1/2 & \oplus & 1/2 \end{array}$$

$$\bar{Q}_{\dot\alpha A} \in (0, 1/2, 1/2) \oplus 1/2 \oplus -1/2$$

$$S^\alpha_A \in (1/2, 0, -1/2) \oplus 1/2 \oplus -1/2$$

$$\bar{S}^{\dot\alpha A} \in (0, 1/2, -1/2) \oplus 1/2 \oplus 1/2.$$

Let $|j, \bar{j}, \Delta, R, r\rangle^{LW} \equiv |\Lambda\rangle^{LW}$

and let $|\Lambda\rangle_a$ be a basis for

$$\mathrm{Span}\left\{ \prod_{\mu,\nu=0,1,2,3}^{3} \rho(M_{\mu\nu})^{m_{\mu\nu}} \, \rho(R_+)^{n} \, |\Lambda\rangle^{LW} \right\}$$

is repn space for $\rho|_h$ $\quad g \supset h = so(3,1) \oplus so(1,1) \oplus su(2)_R \oplus u(1)_r$.

<u>def</u>:
$|\Lambda\rangle_a$ are conformal primaries if

$$\rho(K_\mu)|\Lambda\rangle_a = 0 \quad , \quad \rho(D)|\Lambda\rangle_a = \Delta|\Lambda\rangle_a .$$

The full g_0 repn space is given by Verma module

$$V^{R,r}_{j,\bar{j},\Delta} = \mathrm{Span}\left\{ \prod_{\mu=0}^{3} \rho(P_\mu)^{n_\mu} |\Lambda\rangle_a \right\}$$

Corresponding g-module induced from g_0-module

Since $[D, S^\alpha_A] = -\frac{1}{2} S^\alpha_A$, $[D, \bar{S}^{\dot{\alpha}A}] = -\frac{1}{2}\bar{S}^{\dot{\alpha}A}$

Lower conformal weight.

<u>Def</u> $|\Lambda\rangle_a$ are superconformal primaries if they are conformal primaries and

$$\tilde{\rho}(S^\alpha_A)|\Lambda\rangle_a = \tilde{\rho}(\bar{S}^{\dot{\alpha}A})|\Lambda\rangle_a = 0 .$$

where $\tilde{\rho} \equiv \mathrm{Ind}^{g}_{g_0}\rho$

∴ The induced $\mathfrak{g}$-module is

$$\tilde{\mathcal{V}}^{R,r}_{j,\bar{j},\Delta} = \mathrm{Span}\left\{ \prod_{\mu=0}^{3} \prod_{A,\alpha,\dot{\alpha}=1}^{2} \tilde{\rho}(P_\mu)^{n_\mu} \tilde{\rho}(Q^A_\alpha)^{n_{\alpha A}} \tilde{\rho}(\bar{Q}_{\dot{\alpha}A})^{\bar{n}_{\dot{\alpha}A}} |\Lambda z_a\rangle \right\}$$

<u>Theorem</u> (Dobrev + Petkova '84)

$\tilde{\mathcal{V}}^{R,r}_{j,\bar{j},\Delta}$ is irreducible iff all of the following conditions are false: $\Delta \pm j \pm \bar{j} \in \mathbb{Z}$

$\Delta \pm (j-\bar{j}) \in \mathbb{Z} \setminus \{0\}$

⋮ 10 more.

<u>Theorem</u> (Dobrev + Petkova '85) &

Let $\tilde{\mathcal{V}}^{R,r}_{j,\bar{j},\Delta}$ be irreducible, then $\tilde{\mathcal{V}}^{R,r}_{j,\bar{j},\Delta}$ is unitary iff

$j, \bar{j} \neq 0 \quad \Delta \geq 2+2j+2R+r, \ \Delta \geq 2+2\bar{j}+2R-r$

$j=0 \quad \Delta \geq 2+2\bar{j}+2R-r$

$\bar{j}=0 \quad \Delta \geq 2+2j+2R+r$

<u>Short multiplets</u>

For generic labels $\tilde{\mathcal{V}}^{R,r}_{j,\bar{j},\Delta}$ are general / long

~~∃ several short~~ / atypical ~~reps~~.

In long repns one can, generically, act w/ 8 distinct supercharges Q_α^A, $\bar{Q}_{\dot{\alpha}A}$ obtaining non-trivial states. In these repns

$$\Delta = \Delta(\lambda)$$

$\exists$ Short multiplets. Quotient of generic repn by a non-trivial submodule.

A certain fraction of supercharges will annihilate $|\Lambda\rangle^{LW}$. $\Rightarrow$ Δ related to other labels and will be protected from quantum corrections.

Short repns saturate unitarity bounds.

$\underline{\mathcal{E}_{r,(0,\bar{j})}}$ $R = j = 0$, $\Delta = r$

The lowest weight state $|\Lambda\rangle^{LW}$

Annihilated by

$\tilde{\rho}(Q^{\hat{A}}_{\alpha})|\Lambda\rangle^{LW} = 0$.

"Contains Coulomb branch operators"

Unitarity $\Rightarrow$ $r \geq \bar{j} + 1$

$\underline{\hat{B}_R}$ $j = \bar{j} = r = 0$, $\Delta = 2R$.

Lowest weight state:

$\tilde{\rho}(Q^{A=2}_{\alpha})|\Lambda\rangle^{LW} = \tilde{\rho}(\bar{Q}_{\dot{\alpha}A=1})|\Lambda\rangle^{LW} = 0$.

"Contains Higgs Branch ops".

$\underline{\hat{C}_{R,(j,\bar{j})}}$ $r = \bar{j} - j$, $\Delta = 2R + \bar{j} + j + 2$.

In Particular: $\hat{C}_{0,(0,0)}$ $D|\Lambda\rangle^{LW} = 2|\Lambda\rangle^{LW}$

$$\underbrace{\tilde{\rho}(\bar{Q}_{\dot{\alpha}A})\tilde{\rho}(\bar{Q}_{\dot{\beta}A})}_{A=1}|\Lambda\rangle^{LW} = \underbrace{\tilde{\rho}(Q^{B}_{\alpha})\,\tilde{\rho}(Q^{B}_{\beta})}_{B=2}|\Lambda\rangle^{LW} = 0.$$

Multiplet contains spin 2 conserved current T. w/ $\Delta_T = d = 4$ $\Rightarrow$ Stress tensor multiplet.

full list: hep-th/1412.7131 App B.