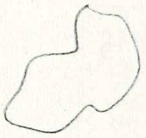
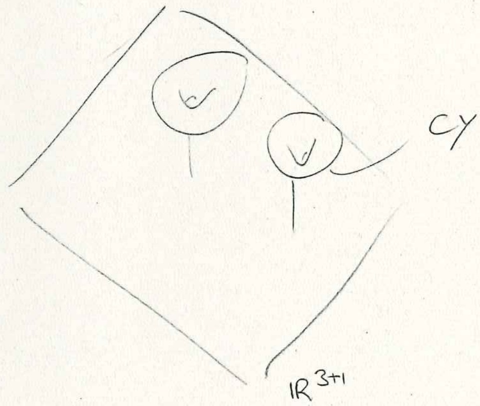
D-branes on CY-manifolds



The (type II) Superstring Theory requires 10-dim space time. To model the observed physics, 6 of 10 dim must be invisible (compactified)



Much studied scenario: 6d-space = Calabi-Yau mfd.

- cplx. mfd. X with trivial canonical bdl. which admits a Kähler metric.

The low energy physics is then represented by a $N=2$ Sugra in $d=4$. This theory depends parametrically on the "shape"-parameters (moduli) of $\mathcal{M}$. There are two types of parameters

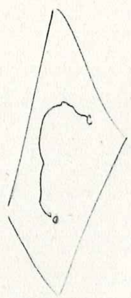
- complex str. of X (periods of holom. 3-form Ω) ($h^{2,1}<\infty$ cplx. params.)
- (complexified) Kähler classes ($h^{1,1}$ params.)

$$u \equiv B+iJ = \sum_e (B_e+iJ_e)e_e .$$

$$\mathcal{M}_X \simeq \mathcal{M}_X^{cplx} \times \mathcal{M}_X^{käh}$$

$e_e \in H^2(X,\mathbb{Z})$ basis

More interesting candidate for particle phys models: $N=1$ Sugra. This arises if space-time contains D-branes.



- Consider closed <u>and</u> open strings, with ends on submfd. of X.

For locus of string-ends in X there exist various possibilities

- free to move in all of X
- restricted to non-contractible submfd. of X (3-dim, 0-dim, 2-dim ...)

Depending on choice of D-brane type one gets different $N=1$ Sugras at low energies.

If, in particular, ends of strings are localized at point in space: D-brane $\simeq$ new type of particle!

Condition of unbroken $N=1$ SUSY
$\Rightarrow$ conditions on D-branes. "BPS-branes"

<u>Example</u>: $\mathrm{Vol}(X)/\ell_s^6 \gg 1$.

BPS-brane: Vector bundle E with connection on X s.t.

a) $F_{ij}\, dz^i dz^j = 0$

b) $g^{j\bar{k}} F_{j\bar{k}\alpha}{}^{\beta} = \mu(E)\,\delta_\alpha^\beta$ (HYM)

or:
$i\hat{F} = F.J = \mu(E)$

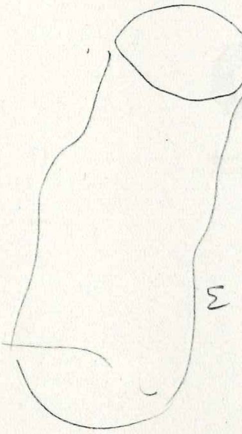
$$\mu(E) = \frac{\deg(E)}{\mathrm{rk}(E)\,\mathrm{Vol}(X)}, \quad \deg(E) = \int_X J \wedge J \wedge c_1(E).$$

Main goal: Classify the BPS-branes for all (Ω, u).

$\Longrightarrow$ Important information on low energy physics (BPS-particles etc.).

Worldsheet-description

For small g_s, l_s one can describe string theory using 2d sigma models.



String motion $\Leftarrow$ minimization of

$$S = \int_\Sigma d^2 z \left(G_{\mu\nu}(x) \frac{\partial x^\mu}{\partial z} \frac{\partial x^\nu}{\partial \bar{z}} + \ldots \right).$$

Here $S = S_{\mathbb{R}^{3+1}} + S_X$. Since X : Calabi-Yau,

$$S_X = -\frac{1}{4\pi\alpha'} \int_\Sigma d^2 z \Big(g_{i\bar{j}} (\partial\phi^i \bar{\partial}\phi^{\bar{j}} + \partial\phi^{\bar{j}} \bar{\partial}\phi^i) + i B_{i\bar{j}} (\partial\phi^i \bar{\partial}\phi^{\bar{j}} - \bar{\partial}\phi^i \partial\phi^{\bar{j}}) + \text{fermions} + \text{boundary terms} \Big)$$

Ricci-flat Kähler metric

(2,2)-SUSY
$\Downarrow$
!

The boundary terms become relevant in presence of D-branes, e.g.

$$\int_{\partial\Sigma} d\tau \, \frac{dx^\mu}{d\tau} \cdot A_\mu(x(\tau))$$

connection on vector bdl. E.

Assuming $\mathrm{Vol}(X)/l_s^6 \gg 1$ such boundary term describes a BPS-brane iff A_μ satisfies (HYM)

More generally: BPS-branes $\leftrightarrow$ boundary conds. that preserve part of $N=(2,2)$ SUSY.

Still assuming large $\mathrm{Vol}(X)$, one can show that there exist two types of BPS-branes:

A-branes: Special Lagrangian submfd. (SLag)

a) $J|_L = 0$ Lagrangian

b) $\Omega|_L = e^{-2\pi i \xi}\,\bar{\Omega}|_L$, $\xi = \text{const.}$

"Special"

B-branes: Solutions of (HYM).

Problem: Only valid for large Vol(X), how to treat general case?

Why hard? If $\mathrm{Vol}\,X \sim l_s^6$ we need all quantum corrs. of Sigma-model.

Strategy to address the Problem was suggested by M. Douglas.

Paradigm: Solution of (HYM) - Donaldson, Yau

1^{st} step: Find solutions of a) $(F^{0,2} = 0)$

$\equiv$ holom. vector bdles E on X

2^{nd} step: Given E, X find necessary and sufficient conds. for solvability of b)

Thm: Solution exists iff E is μ-stable*)

*) For any subbdl. F, $\mu(F) < \mu(E)$.

Recall: $\mu(E) = \dfrac{\deg E}{\mathrm{rk}(E)\,\mathrm{Vol}(X)}$, $\deg E = \int_X J \wedge J \wedge c_1(E)$

Important: Stability depends on J

$\Rightarrow$ Set of stable E may vary with J!

Generalization to stringy regime $\mathrm{Vol}\, X \sim l_s^6$:

1^{st} step: Classify boundary conditions of topologically twisted$^{+)}$ sigma models. (TSM)

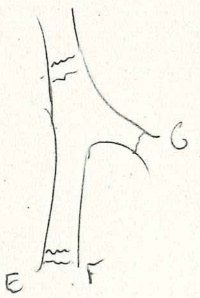
+) Restrict attention to subsector of Σ-model, defd. as cohomology of one of the $N=(2,2)$ supercharges Q.

$\rightsquigarrow$ Subtheory which can be defined rigorously!

It captures important info about full theory ("skeleton").

There are two types of TSM: A/B-model

The result of Step 1 is best formulated in terms of category of D-branes



- objects E, F: topological D-branes, e.g. holom. vector bdl. E
- morphisms Hom(E,F): spaces of top. open strings between E and F.

Proposed result for B-model:

Derived category of coherent sheaves on X

(contains vector bundles on X!)

Proposed result for A-model: Fukaya-cat!

2^{nd} step: Find generalization of stability

- necessary and sufficient conds. for top. D-brane to correspond to a BPS-brane.

The stable D-branes form an abelian subcategory in the category of top. D-branes.

Main ingredient in the proposed conditions for stability is grading $\xi(E)$:

- Generalization of $\mu(E)$ for B-branes.
- Dependent on u (Kähler str.) for B-branes, Ω (cplx. str) for A-branes.
- Phase of central charge
- May be computable via mirror map

	B-model	A-model
top. D-branes	Complexes of coh. sheaves	Lagrangian submfds.
BPS-branes	Π-stable complexes (μ-stability)	Special Lagrangians
Dependence of corr. fcts.	cplx. struct Ω	Kähler str. $B+iJ$
Dependence of stability	Kähler str. $B+iJ$	cplx. struct. Ω